The Analysis of Dispersion for Trajectories of Fire-extinguishing Rocket

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Abstract: In this study a dispersion analysis has been completed on flight simulation software of an unguided fire-extinguishing rocket in order to predict the trajectory parameters and the trajectory dispersion, to analyze the probability of finding the impact point and also to determine the safety area which is a critical condition in firing fields' tests. The total dispersion results mainly due to manufacturing inaccuracy, measurement errors which includes propellant mass, rocket total mass, axial and lateral moments of inertia and position of mass centre, thrust and fin misalignments, errors due to aerodynamic coefficients prediction and atmospheric modelling errors which include atmospheric disturbances such as fluctuations in wind profile, errors of atmospheric properties like pressure, density, temperature.

Key-Words: Dispersion Analysis, Nominal Trajectory, Flight simulation, Fire-extinguishing rocket.

1 Introduction

Dispersion is a measure for the deviation of the rocket's trajectory from the trajectory of some standard rocket, called nominal trajectory. For a rocket, dispersion arises from three different sources: events that occur at launching, events during burning after launching, and events after burning. For rockets, most of the dispersion arises during the burning period after launching. This is predicted theoretically and is confirmed by experience [1].

Simulation of the trajectory of unguided rockets has as input data characteristics of the rocket, atmospheric conditions and launching conditions. In reality all these input data may have some variations around a nominal value that implies a variation in trajectory path. Therefore, there is always an area of possible impact points. The only way to minimize this impact area is to impose some restrictions in manufacturing tolerances, since no restrictions can be made in minimizing the errors of atmospheric conditions. The aim of this paper is to determine the total dispersion of trajectory and to estimate the impact point using the probability analyze method.

The parameters that induce dispersion of rocket's trajectory are:

- The propellant mass and composition inaccuracy;
- The rocket total mass, axial and lateral moments of inertia and resultant centre of gravity inaccuracies;
- Launcher deflection;
- The thrust force of the rocket engine: because of the tolerance in rocket engine design, propellant properties, and manufacturing;
- Thrust and fin misalignments: it is an important source of dispersion in case of unguided rockets.

- Atmospheric disturbances such as wind profile, tail wind, cross wind, and gusts, variation in atmospheric density
- Rocket characteristics, such as aerodynamic coefficients which are previously calculated or measured in wind tunnels.

All these parameters differ from their nominal value and will generate errors derived from measurement, manufacturing or modelling. These sources of error are all mutually independent. Thus, the composite errors are just their combination.

There are some methods to estimate the dispersion of trajectory for a rocket:

- Root Mean Square Method
- Monte Carlo method
- Method of covariance matrix

The Root Mean Square Method simulates the rocket trajectory perturbing one parameter at time and the results are compared with the nominal results. The sum of squares deviations for all parameters is square of total deviation.

Monte Carlo method of dispersion removes smaller dispersion parameters. Each input parameter is selected randomly in the defined ranges and used in the simulation of trajectory.

The Method of covariance matrix: In probability theory and statistics, covariance is a measure of how much two variables change together. A covariance matrix is a matrix whose element in the i, j position is the covariance between the i^{th} and j^{th} elements of a random vector (that is, of a vector of random variables). Each element of the vector is a scalar random variable, either with a finite number of observed empirical values or with a finite or infinite number of potential values specified by a theoretical joint probability distribution of all the random variables.

The parameters uncertainties, based on measurement statistics and observations are presented in table 1.

Table 1

No	Parameter	Variation
1	Mass	±1 %
2	X position of mass centre	±6 %
3	Thrust	±1 %
4	Wind velocity – Ox component	±10 m/s
5	Wind velocity – Oy component	±10 m/s
6	Wind velocity – Oz component	±10 m/s
7	Initial velocity	±2 m/s
8	Moment of inertia	±2%
9	Moment of inertia	±2%
10	Launching angle	±1 deg

All the calculations presented in the paper were performed considering the first 6 parameters variation, but the mathematical model accepts variation for a larger number of parameters that will generate dispersion.

2. Model Description

In order to predict the trajectory of an unguided rocket, six degrees of freedom (6-DOF) mathematical model is used.

As a first approximation for the deviation of trajectory from the nominal value we can investigate the individual influence of one parameter at a time. We'll find some preliminary limits for the trajectory.

Next, we can use possible combinations of parameters to investigate their influence on trajectory. The idea is to ensure that most of the worst possible cases are considered. This is a motivation to investigate the influence of a combination of parameters. The analysis has been done in two ways:

1. For the first model any parameter will pass through its interval of variation with its own defined step; each combination of parameters will generate a new trajectory. The method accuracy strongly depends on step density for every parameter. Also we can associate a probability density value to each parameter individually considering a normal distribution (Gaussian).

In probability theory, the normal (or Gaussian) distribution is a continuous probability distribution that is often used as a first approximation to describe real-valued random variables that tend to cluster around a single mean value. The graph of the associated probability density function is known as the Gaussian function or bell curve (fig. 1):

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

where parameter μ is the mean (location of the peak) and σ^2 is the variance (the measure of the width of the distribution). The probability for the variable to fall within a particular region is given by the integral of this variable's density over the region. The probability density function is maxim around the nominal value of the considered parameter, decreasing to zero near the limits of the interval of variation. The probability density function's integral over the entire space is equal to one.

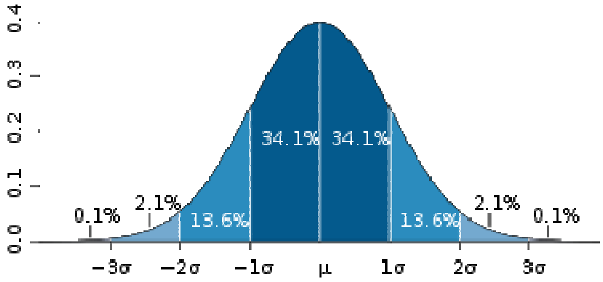

Fig. 1 The graph of Gaussian function

We will obtain a value for the probability density function for every parameter that supports variations. A sum of N independent random variables, with densities U_1, …, U_N:

$$f_{U_1+\ldots+U_N}(x) = \left(f_{U_1} \cdot \ldots \cdot f_{U_N}\right)(x) \qquad (2)$$

2. The second model is based on Monte Carlo Method and consists of generating random values for the variable parameters in the proper interval. Each possible combination of values will generate a new trajectory. This method's accuracy strongly depend on number of iterations, as we can see further in the next section, fig.12 and fig.13.

As in the previous case we can associate a probability density function to each parameter that will generate a probability density function for every point of the trajectory and in particular for the impact point.

The configuration of the rocket has four fins and it is presented in fig. 2. The body of the rocket is cylindrical, with an over-calibre of the front part. A full presentation of this kind of rocket and its utility can be found in [2].

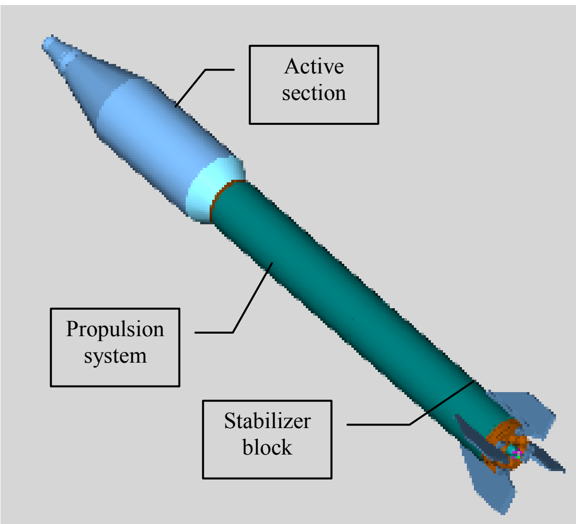

Fig. 2 The Fire-extinguishing Rocket's configuration

3. Simulation results

In this study, a dispersion trajectory calculation using a six-degree-of-freedom (6-DOF) model was developed and applied for fire-extinguishing rocket [3]. All aerodynamic forces and moments coefficients of the configuration are previously calculated, and they are input data. The mass properties are calculated considering the change of the rocket mass during propellant burning till the propellant burn-out (active part), then the rocket will fly the rest of its trajectory as a projectile of fixed mass (passive part). The 6-DOF model assumed the rocket is ideal, where the axis of symmetry of the exterior surface coincides with the longitudinal principal axis of inertia, and the two lateral principal moments of inertia are identical.

3.1. Nominal trajectory

Under the circumstances that no atmospheric disturbances were present, no manufacturing, reading or measuring errors were made, the nominal trajectory have been generated. In order to investigate the trajectory parameters of the given unguided rocket, three cases will be chosen corresponding to firing angles: 30^0, 40^0, 50^0. The 3D representation of nominal trajectories for different launching angles is presented in fig. 3. These cases will give us the possibility to compare the results corresponding to firing angle.

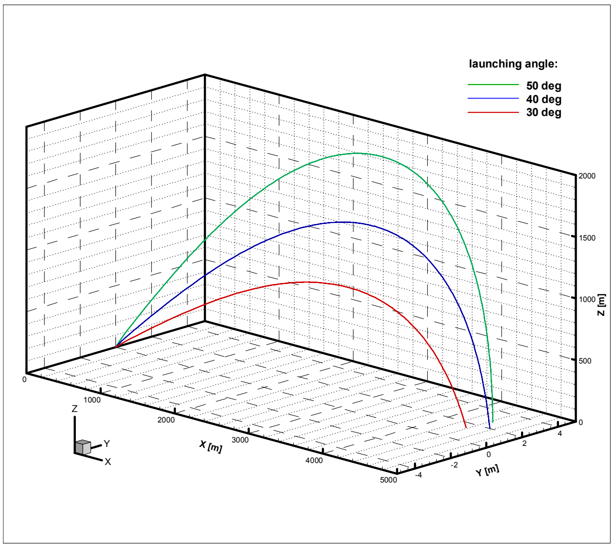

Fig. 3 Nominal trajectories for different launching angles

As we can observe in fig.3 there is a lateral deviation for the nominal trajectories due to rocket rotation.

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3.2 The individual influence of parameters

To investigate the individual influence on trajectory, every parameter will be modified inside the proper variation interval. Using the results it was possible to plot the impact point distance error vs. different parameters variation.

The influence of 1% mass variation on impact point is presented in fig. 4. Errors due to mass centre uncertainty are presented in fig. 5. Figure 6 shows the influence of 1% variation thrust on impact point position. Figure 7 reflects the corresponding deviations of impact point due to wind presence.

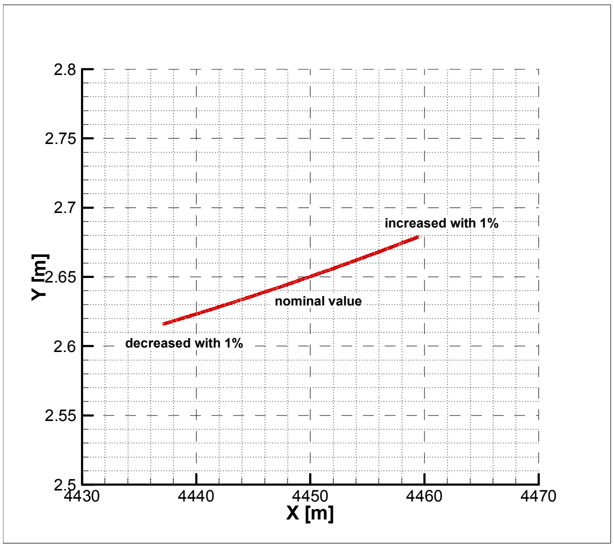

Fig. 4 The influence of mass variation on impact point

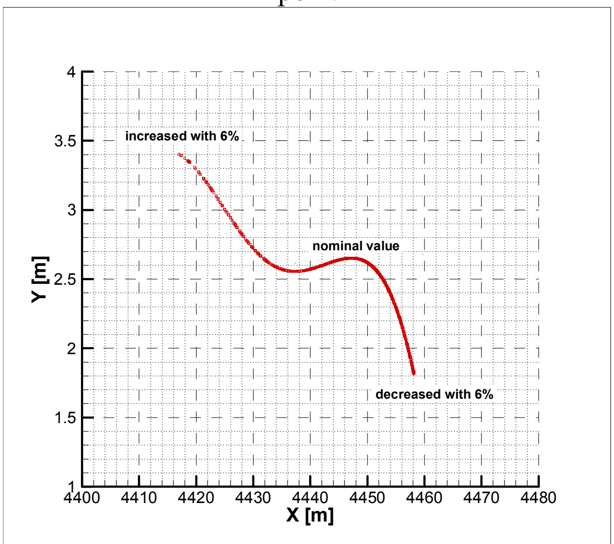

Fig. 5 The influence of mass centre variation on impact point

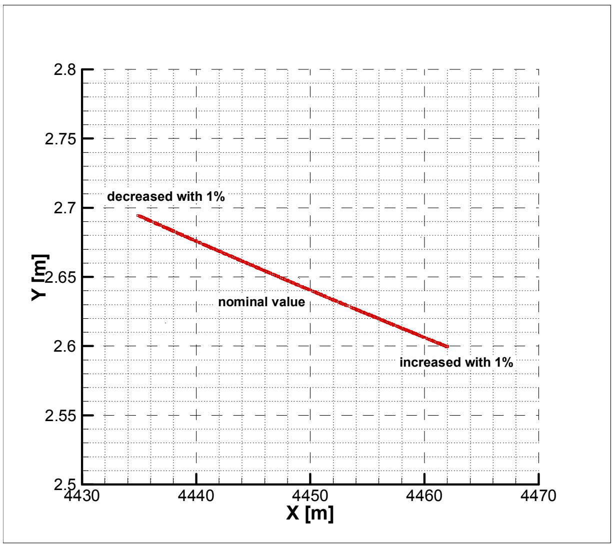

Fig. 6 The effect of the thrust error on impact point

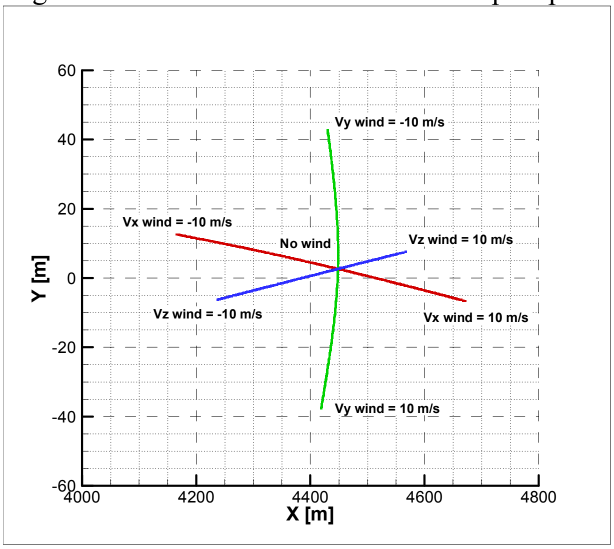

Fig. 7 The effect of the wind velocity on impact point position

The rocket range increased in presence of a tail wind as shown in Fig. 7. Also, if a cross wind is presented the rocket drifts to right if the wind came from right and vice verse due to the presence of tail surfaces behind the rocket centre of gravity to make the rocket fly opposite to wind direction.

Considering all the individual deviations we can obtain an approximation of total deviation. In table 2 there are presented all deviation obtained in the previous example.

Table 2

No	Parameter	Deviation	
		X [m]	Y [m]
1	Mass	22	0.07
2	X position of mass centre	42	1.6
3	Thrust	29	0.11
4	Ox wind velocity	520	22
5	Oy wind velocity	40	82
6	Oz wind velocity	330	18

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The value of total deviation corresponding to the case of 50 deg launching angle is around 630 m.

3.3 The combinations of parameters

In the previous section it was presented an individual analysis of variation for each parameter at once. But combinations of parameters could strongly influence the trajectory path. The results have been obtained using the two models previously described.

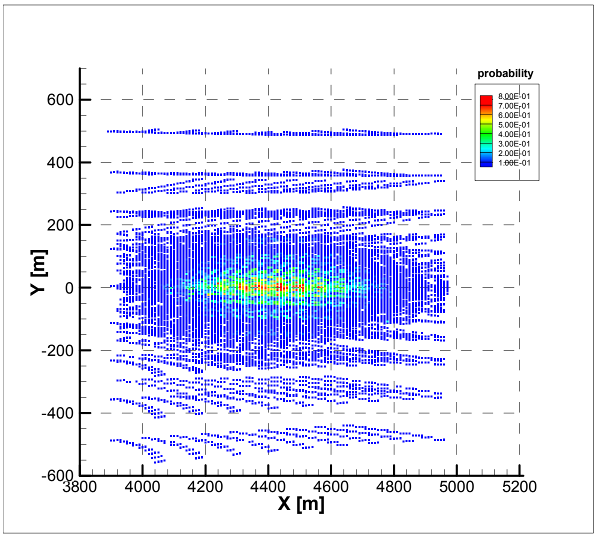

Fig. 8 The area of possible impact point using the first model

The second model used Monte Carlo Method. As we have been expected, the impact probability is higher near the nominal impact point position (fig. 8, 9, 10).

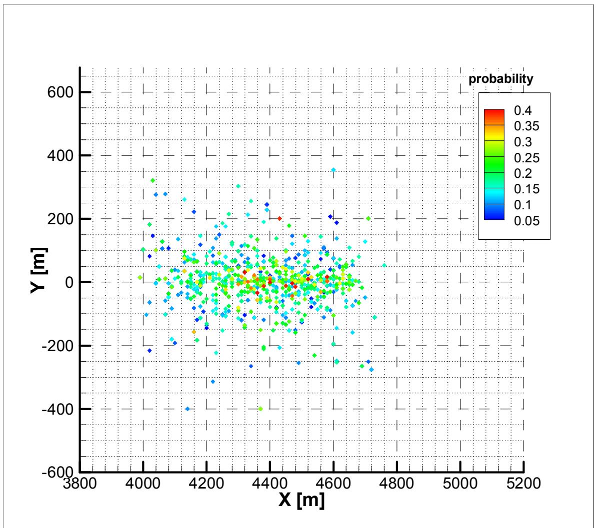

Fig. 9 The area of possible impact point using the Monte Carlo method for 1000 iterations

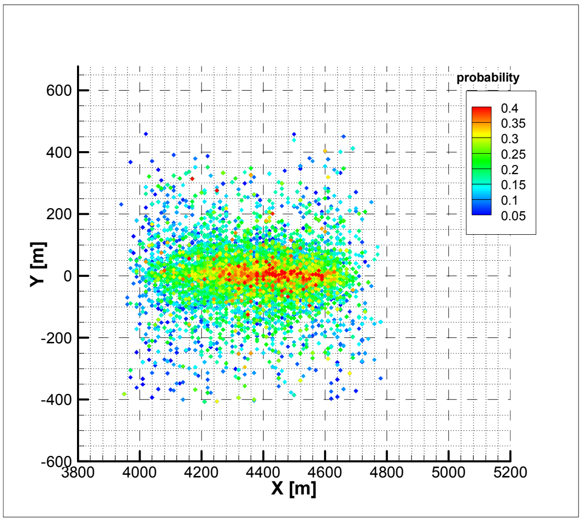

Fig. 10 The area of possible impact point using the Monte Carlo method for 10000 iterations

All the previous graphic results correspond to 50 deg launching angle. Similarly results for 30 deg and 40 deg firing angles have been obtained and they are presented in fig. 11 and 12. The maximum dimensions of dispersion area on impact are presented in table 3.

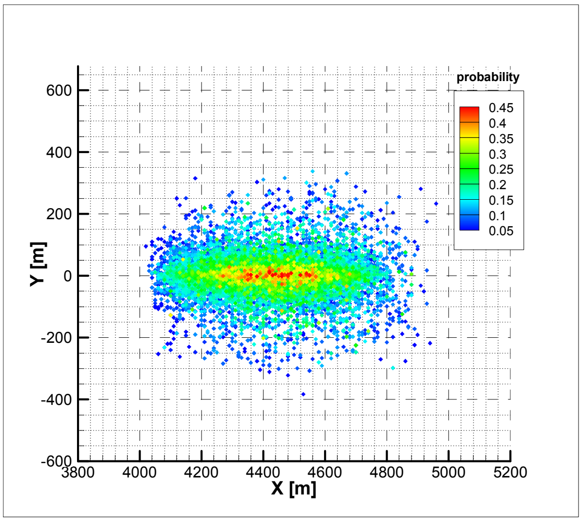

Fig. 11 Dispersion area for 30deg launching angle

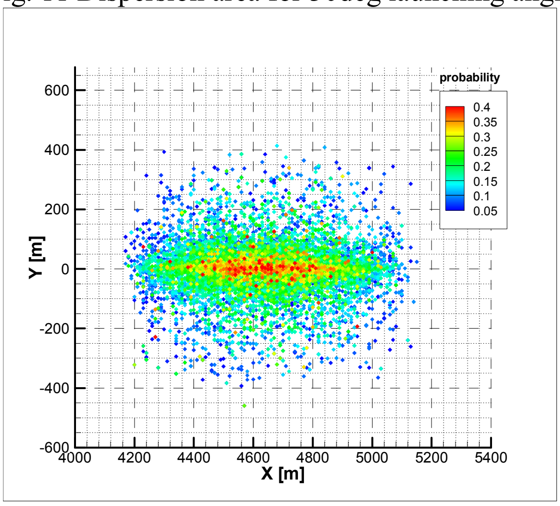

Fig. 12 Dispersion area for 40deg launching angle

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Table 3

Firing angle [deg]	|Xmax-Xmin| [m]	|Ymax-Ymin| [m]
30	930	750
40	1000	810
50	1030	960

From table 3 it is clear that dispersion increases with firing angle because dispersion depends on flight time.

4. Conclusion:

While for some types of rockets is very important to study the dispersion of trajectory at high altitudes (like rockets used to seed the clouds), for the fire-extinguishing rocket the dispersion in impact area is interesting to analyze. The reason is that spreading of extinguishing substances is taking place in the moment of impact. On the other hand, in highland area, for the case of fire localised on slopes, the dispersion on a certain altitude must be estimated. The way to do this is to intersect the fascicle of trajectories with the inclined plane of hill or mountain.

The influence on trajectory of possible combinations of parameters was considered using two different models. The disadvantage of the first model is the expensive computing time, but this model will always capture the extremes. The second model has the advantage of flexibility and rapidity, but it depends on random values and did not exclude the possibility to omit an important case in analysis.

A very important result of this analysis for estimating with high accuracy the possible impact area, in case of normal working rocket is the safety operation of these means of fire extinguishing.

This paper presents the possible dispersion of the fire-extinguishing rocket in case of normal working. We don't take under consideration eventually accidents due to rocket engine malfunction, breaking of some structural or aerodynamic elements or due to a possible explosion during flight.

A similar study can be made to investigate the influence of aerodynamic coefficients uncertainties on the rocket trajectory path and on the impact point.

Because of the results that have been obtaining for three different firing angles we can make a comparison of dispersion for these cases. The conclusion is that dispersion increases with firing angle.

Trajectory analysis of an unguided rocket using simulation software was undertaken to show the importance of this type of analysis in order to know all parameters acting on the rocket during its flight which may be useful to avoid flight mistakes. The dispersion analysis is used to show the importance of identifying the design weaknesses in margins of specific parameters. Also, it is used to find out the optimum values of the rocket parameters for lowest impact point error.

The practical aims of this analysis are:

- Programming of firing tests;
- Finding of rocket's impact point in field to collect the evidences of rocket's efficiency.

Results were presented for the selected conditions in the form of dispersion. This analysis showed that the parameters errors have a great effect on the rocket range and its impact point error.

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